BRST Invariance and renormalisability of the SU(n) Gauge Theory with Massive Gauge Bosons

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Abstract

The problem of renormalisability of the SU(n) theory with massive gauge bosons is reinverstigated in the present work, in view of the insufficient grounds of argument leading to the negative answer in the literature. We expound that the quantization under the Lorentz condition caused by the mass term of the gauge fields leads to a ghost action which is the same as that of the usual SU(n) theory in the Lorentz gauge. We also show that when the $\delta-$functions appearing in the path integral of the Green functions and representing the Lorentz condition are rewritten as Fourier integrals, the BRST invariance is kept in the total effective action consisting of the Lagrange multipliers, ghost fields and the original fields. Furthermore, we clarify that the mass term of the gauge fields cause no additional complexity to the Slavnov-Taylor identity of the generating functional for the regular vertex functions. Finally, with succinct grounds of argument we prove the renormalisability of the theory and reveal that the renormalisability of the theory with the mass term of the gauge fields is ensured by that of the usual SU(n) theory.

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I. Introduction

As is well known, a negative answer to the problem of whether a SU(n) theory with massive gauge bosons is renormalisable was commonly accepted even before knowing the Faddeev-Popov-De Witt method[1−3] to quantized the usual non-Abelian gauge theories and repeatedly maintained in the later literature. Actually the problem has not yet been penetratively analysed. In particular, the Lorentz condition coming from the mass term of the gauge feilds has not been properly treated and thereby some mistakes have been made even in quatization. Recently some colleagues and ourselves have been working to find a positive answer. In this paper we will present our proof and avoid to get entangled in the controversies over differing opinions.

Since the classical equations of motion make the action invariant under an arbitrary infinitesimal transformation of the field functions, they certainly make the mass term of the gauge fields invariant under an arbitrary infinitesimal gauge transformation. Consequently, such a mass term which is not gauge invariant will force the classical equations of motion to produce the Lorentz condition. We will expound that the quantization under the Lorentz condition leads to a ghost action which is the same as that of the usual SU(n) theory in the Lorentz gauge. We will also show that when the $\delta-$ functions appearing in the path integral of the Green functions and representing the Lorentz condition are rewritten as Fourier integrals, the BRST invariance is kept in the total effective action consisting of the Lagrange multipliers λ_a, ghost fields and the original fields.

We will use two kinds of path integral of the generating functional for the Green functions. One of them consists of all the variables including λ_a. Another one is the generating functional for the Green functions in the so-called ξ gauge, which does not involve λ_a. It will be shown that the mass term of the gauge fields cause no extra complexity to the Slavnov-Taylor identity of the generating functional for the regular vertex functions. Consequently, we will be able to determine the general form of the counterterms order by order based on the renormalisability of the usual SU(n) gauge theory and prove that the mass term of the gauge fields is hurmless to the renormalisability of the theory. In this way we will also reveal that the renormalisability of the theory with the mass term of the gauge fields is ensured by that of the usual SU(n) theory.

The method of quantization will be explained in section 2. Setion 3 and section 4 are devoted to prove the renormalisability of the theory. Concluding remarks will be given in the final section.

II. Quantization and BRST Invariance

With $A_{a\mu}$, M standing for the SU(n) gauge fields and their mass parameter the Lagrangian including a mass term $\mathcal{L}_{AM}$ of the gauge fields has the form

$$\mathcal{L} = \mathcal{L}^{(N)} + \mathcal{L}_{AM} \,, \tag{2.1}$$

where

$$\mathcal{L}_{AM} = \frac{1}{2} M^2 A_{a\mu} A_a^\mu \,,$$

$\mathcal{L}$ is the Lagrangian of a usual SU(n) gauge theory and, in the conventional notations, can be written as

$$\begin{aligned} \mathcal{L}^{(N)} &= -\frac{1}{4} F_{a\mu\nu} F_a^{\mu\nu} + \mathcal{L}_\psi + \mathcal{L}_{\psi A} \,, \\ F_{a\mu\nu} &= \partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - g f_{abc} A_{b\mu} A_{c\nu} \,. \end{aligned}$$

Under the infinitesimal gauge transformation, one has

$$\begin{aligned} \delta \int d^4x \mathcal{L}(x) &= \int d^4x M^2 A_{a\mu} \delta A_a^\mu = -\frac{1}{g} M^2 \int d^4x A_{a\mu} \partial^\mu \delta\theta_a \\ &= \frac{1}{g} M^2 \int d^4x \big(\partial^\mu A_{a\mu}\big) \delta\theta_a \,. \end{aligned}$$

where δA_a^μ stands for the infinitesimal gauge transformation of the gauge feilds

$$\delta A_a^\mu = -\frac{1}{g} \partial^\mu \delta\theta_a(x) - f_{abc} \delta\theta_b(x) A_c^\mu(x) \,. \tag{2.2}$$

Since the classical equations of motion make the action invariant under an arbitrary infinitesimal transformation of the field functions, they certainly make the mass term of the gauge fields invariant under an arbitrary infinitesimal gauge transformation. This means that when M is not equal to zero, the classical equations of motion leads to the following Lorentz condition

$$\partial^\mu A_{a\mu} = 0 \,, \tag{2.3}$$

It should be noticed with emphasis that the Lorentz condition makes the mass term invariant with respect to the infinitesimal gauge trasformation. Consequently, the combination of the action and the Lorentz condition is invariant with respect to the infinitesimal gauge trasformation that satisfies the following equations

$$\delta\big(\partial^\mu A_{a\mu}\big) = 0 \,. \tag{2.4}$$

Since such a residual invariance is not broken by the mass term of the gauge fields it is natural to imagine that the ghost action should be the same as that of the usual SU(n) theory in the Lorentz gauge (See for example, Ref.[4]). However, this was not considered in the literature. For intance, in the discussion in [5], concerning the massive gauge fields theory without matter fields, the original form of the generating functional for the Green functions was taken to be

$$\int \mathcal{D}[\mathcal{A}]\exp\{\mathrm{i}\big[I + J_a^\mu(x)A_{a,\mu}(x)\big]\}\,,$$

where $J_a^\mu(x)A_{a,\mu}(x)$ is the source term and I is the action defined by $\mathcal{L}(x)$. In this way, the Lorentz condition (2.3) was ignored.

Taking the Lorentz condition into account one should write the path integral of the Green functions inolving only the original fields as

$$\frac{1}{N_0}\int \mathcal{D}[\mathcal{A},\overline{\psi},\psi]\Delta[\mathcal{A}]\prod_{a',x'}\delta\left(\partial^\sigma A_{a'\sigma}(x')\right)A_{a\mu}(x)A_{b\nu}(y)A_{c\rho}(z)\cdots\exp\{\mathrm{i}I\}\,, \tag{2.5}$$

where

$$N_0 = \int \mathcal{D}[\mathcal{A},\overline{\psi},\psi]\Delta[\mathcal{A}]\prod_{a',x'}\delta\left(\partial^\lambda A_{a'\lambda}(x')\right)\exp\{\mathrm{i}I\}\,.$$

The weight factor $\Delta[\mathcal{A}]$ is to be determnined. Since only the field functions which satisfy the Lorentz condition can play roles in the integral (2.5), the value of the Lagrangian can be changed for the field functions which do not satisfy this condition. In view of the fact that the Lorentz condition makes the mass term invariant with respect to the infinitesimal gauge trasformation, we now imagine to replace the mass term $\mathcal{L}_{AM}$ with a gauge invariant mass term $\widetilde{\mathcal{L}}_{AM}$ which is equal to $\mathcal{L}_{AM}$ when the Lorentz condition is satisfied. Thus, analogous to the case in the Fadeev–Popov method[1,6], $\Delta[\mathcal{A}]$ should be gauge invariant and make the following equation valid for an arbitrary gauge invariant quantity $\mathcal{O}(\mathcal{A},\,\overline{\psi},\,\psi)$

$$\int \mathcal{D}[\mathcal{A},\,\overline{\psi},\,\psi]\Delta[\mathcal{A}]\prod_{a',x'}\delta\left(\partial^\lambda A_{a'\lambda}(x')\right)\mathcal{O}(\mathcal{A},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\} \propto \int \mathcal{D}[\mathcal{A},\,\overline{\psi},\,\psi]\mathcal{O}(\mathcal{A},\overline{\psi},\psi)\exp\{\mathrm{i}\widetilde{I}\}\,.$$

where $\widetilde{I}$ is a gauge invariant action obtained by replacing $\mathcal{L}_{AM}$ with $\widetilde{\mathcal{L}}_{AM}$. This means that the weight factor $\Delta[\mathcal{A}]$ can be determined according to the Fadeev–Popov equation in the usual SU(n) theory under the Lorentz condition. Therfore, when the theory is generalized to include the F–P ghost fields $C_a(x)$ and $\overline{C}_a(x)$, the ghost term in the effective Lagrangian or action has the same form as that of the usual SU(n) theory in the Lorentz gauge. Namely

$$\mathcal{L}^{(C)}(x) = \big(-\partial_\mu\overline{C}_a(x)\big)D^\mu_{ab}C_b(x)\,, \qquad I^{(C)} = \int d^4x\mathcal{L}^{(C)}(x)\,, \tag{2.6}$$

where

$$D^{\mu}_{ab}(x) = \delta_{ab}\partial^{\mu} + g f_{abc} A^{\mu}_{c}(x)\,, \tag{2.7}$$

As usual one can further generalized the theory by regarding as new variables the Lagrange multipliers $\lambda_a(x)$ associated with the Lorentz condition. Thus the total effective Lagrangian and action are

$$\mathcal{L}_{eff}(x) = \mathcal{L} + \mathcal{L}^{(C)}(x) + \lambda_a(x)\partial^{\mu} A_{a\mu}(x)\,, \tag{2.8}$$

$$I_{eff} = \int d^4x \mathcal{L}_{eff}(x)\,. \tag{2.9}$$

Correspondingly, the path integral of the generating functional for the Green functions is

$$\mathcal{Z}[\overline{\eta}, \eta, \overline{\chi}, \chi, J, j] = \frac{1}{N_{\lambda}} \int \mathcal{D}[\mathcal{A}, \overline{\psi}, \psi, \overline{C}, C, \lambda] \exp\Big\{ \mathrm{i}\big(I_{\text{eff}} + I_s\big)\Big\}\,, \tag{2.10}$$

where N_{λ} is a constant to make $\mathcal{Z}[0,0,0,0,0,0]$ equal to 1, I_s is the source term in the action. They are defined by

$$\begin{aligned} N_{\lambda} &= \int \mathcal{D}[\mathcal{A}, \overline{\psi}, \psi, \overline{C}, C, \lambda] \exp\Big\{ \mathrm{i} I_{\text{eff}}\Big\}\,, \\ I_s &= \int d^4x \big[J^{\mu}_a(x) A_{a\mu}(x) + j_a(x)\lambda_a(x) + \overline{\chi}_a(x) C_a(x) \\ &\qquad + \overline{C}_a(x)\chi_a(x) + \overline{\eta}_a(x)\psi_a(x) + \overline{\psi}_a(x)\eta_a(x)\big]\,, \end{aligned}$$

where $J^{\mu}_a(x)$, $j_a(x)$, $\overline{\chi}_a(x)$, $\chi_a(x)$ and $\overline{\eta}$, η are the sources associate to various fields. It is understood as usual that in a direct calculation of the path integral, the effective Lagrangian $\mathcal{L}_{eff}$ contains some additional iϵ terms.

We now check the BRST invariance of the effective action I_{eff} defined by (2.8) and (2.9). With the gauge fields, the matter fields and the ghost fields transforming as usual, one has

$$\begin{aligned} &\delta_B A^{\mu}_a = \delta\zeta D^{\mu}_{ab} C_b(x)\,, \\ &\delta_B \overline{C}_a(x) = -\delta\zeta \lambda_a(x)\,, \\ &\delta_B C_a(x) = \delta\zeta \frac{g}{2} f_{abc} C_b(x) C_c(x)\,, \\ &\delta_B I_{eff} = \int d^4x \Big\{ \big(\delta_B \lambda_a(x) - \delta\zeta M^2 C_a(x)\big)\partial^{\mu} A_{a\mu}\Big\}\,. \end{aligned}$$

where $\delta\zeta$ is an infinitesimal fermionic parameter independent of x. Obviously, the effective action is invariant when the transformation of $\lambda_a(x)$ are defined as

$$\delta_B \lambda_a(x) = \delta\zeta M^2 C_a(x)\,.$$

It is also clear that the transformation is nolonger nilpotent.

We are also interested in the ξ gauge Green functions that are defined by replacing the $\delta-$functions in the numerator and denominator of (2.5) with the gauge-fixing term

$$-\frac{1}{2\xi}(\partial^{\mu}A_{a\mu})^2 \,.$$

where ξ is a pararmter. The total effective Lagrangian and action including the gauge-fixing term and the ghost term become (in the same notations as used above)

$$\mathcal{L}_{eff}(x) = \mathcal{L} + \mathcal{L}^{(C)}(x) - \frac{1}{2\xi}(\partial^{\mu}A_{a\mu})^2 \,, \tag{2.11}$$

$$I_{\text{eff}} = \int d^4x \mathcal{L}_{\text{eff}}(x) \,. \tag{2.12}$$

Therefore the generating functional for such Green functions is

$$\mathcal{Z}[\overline{\eta}, \eta, \overline{\chi}, \chi, J] = \frac{1}{N_\xi}\int \mathcal{D}[\mathcal{A}, \overline{\psi}, \psi, \overline{C}, C] A_{a\mu}(x) A_{b\nu}(y) A_{b\rho}(z) \cdots \exp\Big\{ \mathrm{i} I_{\text{eff}} \Big\} \,, \tag{2.13}$$

where N_ξ is a constant to make $\mathcal{Z}[0,0,0,0,0]$ equal to 1, I_s is the source term

$$I_s = \int d^4x \big[J^{\mu}_a(x) A_{a\mu}(x) + \overline{\chi}_a(x) C_a(x) + \overline{C}_a(x)\chi_a(x) + \overline{\eta}_a(x)\psi_a(x) + \overline{\psi}_a(x)\eta_a(x) \big] \,,$$

It shuld be noticed that the Lorentz condition will take no effect in the generating functional for the ξ guage Green functions unless ξ tends to zero.

III. Renormalisability

As stated earlier, for a long period of time a SU(n) theory with massive gauge bosons was considered to be in general nonrenormalisable (See for example [5,7–11]) and the grounds were actually insufficient for drawing this conclusion. Based on the quantization method explained in last section we will prove the renormalisability of the theory. In this section we will start with the the generating functional for the ξ gauge Green functions. The method of reasoning for the theory formalism with the variables λ_a is similar and will be briefly described in section 4.

Assume that $A_{a\mu}(x)$, $C_a(x)$ and $\overline{C}_a(x)$ stand for the renormalized feild founctions, g, M are renormalized parameters, and ξ is an auxiliary parameter. The matter feilds ψ, $\overline{\psi}$ do not affect the discussion in this section and will be omitted. As usual we define the composite field functions $\Delta A^{\mu}_a(x)$ and $\Delta C_a(x)$ by

$$\delta_B A^{\mu}_a(x) = \delta\zeta \Delta A^{\mu}_a(x) \,, \qquad \delta_B C_a(x) = \delta\zeta \Delta C_a(x) \,, \tag{3.1}$$

where $\Delta A^{\mu}_a(x)$ is just $D^{\mu}_{ab}C_b(x)$ and $\Delta C_a(x)$ is $\frac{1}{2}gf_{abc}C_b(x)C_c(x)$. Introducing new soruces $K^a_\mu(x)$ and $L_a(x)$ and adding a soruce term of these composite fields into the effective Lagranian without counterterm, one gets

$$\begin{aligned}\mathcal{L}^{[0]}_{\text{eff}}(x) = & - \frac{1}{4}F_{a\mu\nu}(x)F^{\mu\nu}_a(x) + \frac{1}{2}M^2 A_{a\mu}(x)A^{\mu}_a(x) - \frac{1}{2\xi}\Big(\partial^\nu A_{a\nu}(x)\Big)^2 \\ & + \big(-\partial_\mu \overline{C}_a(x)\big)D^{\mu}_{ab}C_b(x) \\ & + K^a_\mu(x)\Delta A^\mu_a(x) + L_a(x)\Delta C_a(x)\,. \end{aligned} \tag{3.2}$$

The complete effective Lagranian is the sum of $\mathcal{L}^{[0]}_{\text{eff}}$ and the counterterm $\mathcal{L}_{count}$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{[0]}_{\text{eff}} + \mathcal{L}_{count}\,. \tag{3.3}$$

In terms of the action $I^{[0]}_{eff}$ formed by the effective Lagranian $\mathcal{L}^{[0]}_{eff}$, we define the generating functional for Green functions

$$\mathcal{Z}^{[0]}[J,\overline{\chi},\chi,K,L] = \frac{1}{N}\int \mathcal{D}[\mathcal{A},\overline{C},C]\exp\Big\{\mathrm{i}\big(I^{[0]}_{\text{eff}} + I_s\big)\Big\}\,, \tag{3.4}$$

where N is a constant to make $\mathcal{Z}^{[0]}[0,0,0,0,0]$ equal to 1, the source term I_s is given by

$$I_s = \int d^4x\big[J^\mu_a(x)A_{a\mu}(x) + \overline{\chi}_a(x)C_a(x) + \overline{C}_a(x)\chi_a(x)\big]\,,$$

Correspondingly, the generating functionals $\mathcal{W}^{[0]}$, $\Gamma^{[0]}$ for connected Green functions and regular vertex functions are

$$\mathcal{Z}^{[0]}[J,\overline{\chi},\chi,K,L] = \exp\Big\{\mathrm{i}\mathcal{W}^{[0]}[J,\overline{\chi},\chi,K,L]\Big\}\,, \tag{3.5}$$

$$\begin{aligned}\Gamma^{[0]}[\widetilde{A},\widetilde{\overline{C}},\widetilde{C},K,L] = & \mathcal{W}^{[0]}[J,\overline{\chi},\chi,K,L] \\ & - \int d^4x\Big[J^\mu_a(x)\widetilde{A}_{a\mu}(x) + \overline{\chi}_a(x)\widetilde{C}_a(x) + \widetilde{\overline{C}}_a(x)\chi_a(x)\Big]\,,\end{aligned} \tag{3.6}$$

where $\widetilde{A}_{a\mu}(x)$, $\widetilde{C}_a(x)$ and $\widetilde{\overline{C}}_a(x)$ are the so-called classical fields defined by

$$\widetilde{A}_{a\mu}(x) = \frac{\delta\mathcal{W}^{[0]}}{\delta J^\mu_a(x)}\,, \qquad \widetilde{C}_a(x) = \frac{\delta\mathcal{W}^{[0]}}{\delta\overline{\chi}_a(x)}\,, \qquad \widetilde{\overline{C}}_a(x) = -\frac{\delta\mathcal{W}^{[0]}}{\delta\chi_a(x)}\,. \tag{3.7}$$

One therefore has

$$J^\mu_a(x) = -\frac{\delta\Gamma^{[0]}}{\delta\widetilde{A}_{a\mu}(x)}\,, \qquad \overline{\chi}_a(x) = \frac{\delta\Gamma^{[0]}}{\delta\widetilde{C}_a(x)}\,, \qquad \chi_a(x) = -\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_a(x)}\,, \tag{3.8}$$

and

$$\frac{\delta\mathcal{W}^{[0]}}{\delta K^a_\mu(x)} = \frac{\delta\Gamma^{[0]}}{\delta K^a_\mu(x)}\,, \qquad \frac{\delta\mathcal{W}^{[0]}}{\delta L_a(x)} = \frac{\delta\Gamma^{[0]}}{\delta L_a(x)}\,, \tag{3.9}$$

In order to find the Slavnov–Taylor identity satisfied by the generating functional for the proper functions, we change the variables in the path integral of $\mathcal{Z}^{[0]}$ as follows

$$
\begin{aligned}
&A_a^\mu(x) \to A_a^\mu(x) + \delta\zeta \Delta A_a^\mu(x) \,, \\
&C_a(x) \to C_a(x) + \delta\zeta \Delta C_a(x) \,, \\
&\overline{C}_a(x) \to \overline{C}_a(x) + \delta\zeta \frac{1}{\xi}\partial_\mu A_a^\mu(x) \,.
\end{aligned}
$$

The volume element of the path integral does not change and the changes of the source term and the mass term of the gauge fields lead to

$$
\begin{aligned}
\int d^4x \Big\{ & \frac{\delta\Gamma^{[0]}}{\delta K_\mu^a(x)} \frac{\delta\Gamma^{[0]}}{\delta \widetilde{A}_a^\mu(x)} + \frac{\delta\Gamma^{[0]}}{\delta L_a(x)} \frac{\delta\Gamma^{[0]}}{\delta \widetilde{C}_a(x)} \\
& + \frac{1}{\xi}\big(\partial_\mu \widetilde{A}_a^\mu(x)\big) \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} - M^2 \widetilde{A}_{a\mu}(x) \frac{\delta\Gamma^{[0]}}{\delta K_\mu^a(x)} \Big\} = 0 \,,
\end{aligned}
\tag{3.10}
$$

Next, by using the invariance of the path integral of $\mathcal{Z}^{[0]}$ with respect to the translation of the integration variables $\overline{C}_a(x)$, one can get a set of auxiliary identities

$$
\partial_\mu \frac{\delta\Gamma^{[0]}}{\delta K_\mu^a(x)} - \frac{\delta\Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} = 0 \,,
\tag{3.11}
$$

In the following we will denote by $\Gamma^{[0]}[A, \overline{C}, C, K, L]$ the functional that is obtained from $\Gamma^{[0]}[\widetilde{A}, \widetilde{\overline{C}}, \widetilde{C}, K, L]$ by replacing the classical field functions with the usual field functions. Defined $\overline{\Gamma}^{[0]}$ as

$$
\overline{\Gamma}^{[0]} = \Gamma^{[0]} + \int d^4x \Big\{ \frac{1}{2\xi}\Big(\partial^\nu A_{a\nu}(x)\Big)^2 \Big\} - \int d^4x \Big\{ \frac{1}{2} M^2 A_{a\mu}(x) A_a^\mu(x) \Big\} \,,
\tag{3.12}
$$

Then (3.10) and (3.11) lead to

$$
\int d^4x \Big\{ \frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^a(x)} \frac{\delta\overline{\Gamma}^{[0]}}{\delta A_a^\mu(x)} + \frac{\delta\overline{\Gamma}^{[0]}}{\delta L_a(x)} \frac{\delta\overline{\Gamma}^{[0]}}{\delta C_a(x)} \Big\} = 0 \,,
\tag{3.13}
$$

$$
\partial_\mu \frac{\delta\overline{\Gamma}^{[0]}}{\delta K_\mu^a(x)} - \frac{\delta\overline{\Gamma}^{[0]}}{\delta \overline{C}_a(x)} = 0 \,.
\tag{3.14}
$$

Assume that the dimensional regularization method is used and the relations (3.13) and (3.14) are guaranteed. Denote the tree part and one loop part of $\overline{\Gamma}^{[0]}$ by $\overline{\Gamma}_0^{[0]}$ and $\overline{\Gamma}_1^{[0]}$ respectively, $\overline{\Gamma}_0^{[0]}$ is thus the modified action $\overline{I}_{eff}^{[0]}$ without the gauge fixing term and the mass term of the gauge fields. From (3.13), (3.14) one has

$$
\overline{\Gamma}_0^{[0]} * \overline{\Gamma}_0^{[0]} = 0 \,,
\tag{3.15}
$$

$$
\partial_\mu \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K_\mu^a(x)} - \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta \overline{C}_a(x)} = 0 \,.
\tag{3.16}
$$

and

$$\overline{\Gamma}_0^{[0]} * \overline{\Gamma}_1^{[0]} + \overline{\Gamma}_1^{[0]} * \overline{\Gamma}_0^{[0]} = \Lambda_{\rm op}\overline{\Gamma}_1^{[0]} = 0\,, \tag{3.17}$$

$$\partial_\mu \frac{\delta\overline{\Gamma}_1^{[0]}}{\delta K_\mu^a(x)} - \frac{\delta\overline{\Gamma}_1^{[0]}}{\delta\overline{C}_a(x)} = 0\,. \tag{3.18}$$

The notations $A * B$, $\Lambda_{\rm op}$ are defined in the usual way, namely

$$A * B = \int d^4x \Big\{ \frac{\delta A}{\delta K_\mu^a(x)} \frac{\delta B}{\delta A_{a\mu}(x)} + \frac{\delta A}{\delta L_a(x)} \frac{\delta B}{\delta C_a(x)} \Big\}\,, \tag{3.19}$$

$$\Lambda_{\rm op} = \int d^4x \Big\{ \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K_\mu^a(x)} \frac{\delta}{\delta A_a^\mu(x)} + \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta A_a^\mu(x)} \frac{\delta}{\delta K_\mu^a(x)} + \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta L_a(x)} \frac{\delta}{\delta C_a(x)} + \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_a(x)} \frac{\delta}{\delta L_a(x)} \Big\}\,, \tag{3.20}$$

The pole part of $\overline{\Gamma}_1^{[0]}$ will be denoted by $\overline{\Gamma}_{1,{\rm div}}^{[0]}$. Of course it also satisfies (3.17) and (3.18), namely

$$\Lambda_{\rm op}\overline{\Gamma}_{1,{\rm div}}^{[0]} = 0\,, \tag{3.21}$$

$$\partial_\mu \frac{\delta\overline{\Gamma}_{1,{\rm div}}^{[0]}}{\delta K_\mu^a(x)} - \frac{\delta\overline{\Gamma}_{1,{\rm div}}^{[0]}}{\delta\overline{C}_a(x)} = 0\,, \tag{3.22}$$

This is the same equations as appears in the usual SU(n) theory.

If $M = 0$ then it is known from the renormalisability of the theory that $\overline{\Gamma}_{1,{\rm div}}^{[0]}$ is a combination of the three terms

$$g\frac{\partial\overline{\Gamma}_0^{[0]}}{\partial g}\,, \qquad \int d^4x \Big\{ A_{a\nu}(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta A_{a\nu}(x)} + L_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta L_a(x)} \Big\}\,,$$

$$\int d^4x \Big\{ C_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_a(x)} + \overline{C}_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{C}_a(x)} + K_\mu^a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K_\mu^a(x)} \Big\}\,,$$

Since each of these satisfies equations (3.21),(3.22) a new term appearing when $M \neq 0$, if any, should includ M^2 as a factor and also satisfies (3.21), (3.22). Now the equations can not have such a solution. It follows that

$$\begin{aligned}\overline{\Gamma}_{1,{\rm div}}^{[0]} = \alpha_1 \Big(g\frac{\partial\overline{\Gamma}_0^{[0]}}{\partial g} \Big) + \beta_1 \int d^4x \Big\{ A_{a\nu}(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta A_{a\nu}(x)} + L_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta L_a(x)} \Big\} \\ +\gamma_1 \int d^4x \Big\{ C_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta C_a(x)} + \overline{C}_a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta\overline{C}_a(x)} + K_\mu^a(x) \frac{\delta\overline{\Gamma}_0^{[0]}}{\delta K_\mu^a(x)} \Big\}\,.\end{aligned} \tag{3.23}$$

where α_1, β_1 and γ_1 are constants of order $(\hbar)^1$ and are divergent when the space-time dimention of tens to 4.

In order to cancel the one loop divergence the counterterm of order $\hbar^1$ in the action should be chosen as

$$\delta I_{count}^{[1]}[A, C, \overline{C}, K, L, g, M] = -\overline{\Gamma}_{1,{\rm div}}^{[0]}[A, C, \overline{C}, K, L, g, M]\,. \tag{3.24}$$

Thus the sum of $\overline{\Gamma}_0^{[0]}$ and $\delta I^{[1]}_{count}$, to order of $\hbar^1$, can be written as

$$\begin{aligned}
&\overline{I}^{[1]}_{\rm eff}[A, C, \overline{C}, K, L, g] \\
&\qquad = \overline{\Gamma}_0^{[0]}[(Z_3^{[1]})^{1/2}A, (\widetilde{Z}_3^{[1]})^{1/2}C, (\widetilde{Z}_3^{[1]})^{1/2}\overline{C}, (\widetilde{Z}_3^{[1]})^{1/2}K, (Z_3^{[1]})^{1/2}L, Z_g^{[1]}g]\,, \qquad (3.25)
\end{aligned}$$

where

$$\begin{aligned}
(Z_3^{[1]})^{1/2} &= 1 - \beta_1\,, \\
(\widetilde{Z}_3^{[1]})^{1/2} &= 1 - \gamma_1\,, \\
Z_g^{[1]} &= 1 - \alpha_1\,.
\end{aligned}$$

Next by adding the gaue fixing term and the mass term of the gauge fields into $\widehat{I}^{[1]}_{\rm eff}]$ and forming

$$\begin{aligned}
I^{[1]}_{eff}[A, C, \overline{C}, K, L, g, M, \xi] \quad &= \quad \overline{I}^{[1]}_{\rm eff}[A, C, \overline{C}, K, L, g] \\
&\quad - \int d^4x \Big\{ \frac{1}{2\xi}\Big(\partial^\nu A_{a\nu}(x)\Big)^2 \Big\} \\
&\quad + \int d^4x \Big\{ \frac{1}{2} M^2 A_{a\mu}(x) A_a^\mu(x) \Big\}\,, \qquad (3.26)
\end{aligned}$$

one has

$$I^{[1]}_{eff}[A, C, \overline{C}, K, L, g, M, \xi] = I^{[0]}_{eff}[A^{[0]}, C^{[0]}, \overline{C}^{[0]}, K^{[0]}, L^{[0]}, g^{[0]}, M^{[0]}, \xi^{[0]}]\,, \qquad (3.27)$$

where $A^{[0]}, C^{[0]}, \overline{C}^{[0]}, \cdots$, to order $\hbar^1$, stand for the bare quantities and are defined by

$$A^{[0]}_{a\mu} = (Z_3^{[1]})^{1/2} A_{a\mu}, \quad C^{[0]}_a = (\widetilde{Z}_3^{[1]})^{1/2} C_a, \quad \overline{C}^{[0]}_a = (\widetilde{Z}_3^{[1]})^{1/2} \overline{C}_a\,, \qquad (3.28)$$

$$K^{[0]a}_\mu = (\widetilde{Z}_3^{[1]})^{1/2} K^a_\mu, \quad L^{[0]}_a = (Z_3^{[1]})^{1/2} L_a\,, \qquad (3.29)$$

$$g^{[0]} = Z_g^{[1]} g, \quad M^{[0]} = (Z_3^{[1]})^{-1/2} M, \quad \xi^{[0]} = Z_3^{[1]}\xi\,. \qquad (3.30)$$

Obviously, if the action $I^{[1]}_{eff}[A, C, \overline{C}, K, L, g, M, \xi]$ is used to replace $I^{[0]}_{eff}[A, C, \overline{C}, K, L, g, M, \xi]$ in (3.4) and define the generating functiomal $\Gamma^{[1]}$ as well as

$$\overline{\Gamma}^{[1]} = \Gamma^{[1]} + \int d^4x \Big\{ \frac{1}{2\xi}\Big(\partial^\nu A_{a\nu}(x)\Big)^2 \Big\} - \int d^4x \Big\{ \frac{1}{2} A_a^\mu(x) A_{a\mu}(x) \Big\}\,,$$

then one has

$$\overline{\Gamma}^{[1]}[A, \overline{C}, C, K, L] = \overline{\Gamma}^{[0]}[A^{[0]}, \overline{C}^{[0]}, C^{[0]}, K^{[0]}, L^{[0]}]\,. \qquad (3.31)$$

We then expand the right hand side of this equation into the form

$$\overline{\Gamma}_0^{[0]}[A^{[0]}, \overline{C}^{[0]}, C^{[0]}, K^{[0]}, L^{[0]}] + \overline{\Gamma}_1^{[0]}[A^{[0]}, \overline{C}^{[0]}, C^{[0]}, K^{[0]}, L^{[0]}] + \cdots$$

In the first term the divergences of order $\hbar^1$ are due to $\delta I^{[1]}_{count}$. In the second term the divergences of this order do not contain the contribution of $\delta I^{[1]}_{count}$ and are therefore due to the action of order $\hbar^0$. It follows that, to order $\hbar^1$, $\widehat{\Gamma}^{[1]}$ is finite. Moreover from (3.13), (3.14) and (3.31) one gets

$$\int d^4x \Big\{ \frac{\delta\overline{\Gamma}^{[1]}}{\delta(K^{[0]})^a_\mu(x)} \frac{\delta\overline{\Gamma}^{[1]}}{\delta A^{[0]}_{a\mu}(x)} + \frac{\delta\overline{\Gamma}^{[1]}}{\delta L^{[0]}_a(x)} \frac{\delta\overline{\Gamma}^{[1]}}{\delta C^{[0]}_a(x)} \Big\} = 0\,, \tag{3.32}$$

$$\partial_\mu \frac{\delta\overline{\Gamma}^{[1]}}{\delta(K^{[0]})^a_\mu(x)} - \frac{\delta\overline{\Gamma}^{[1]}}{\delta\overline{C}^{[0]}_a(x)} = 0\,. \tag{3.33}$$

With the help of (3.28)–(3.30), these equations can written as

$$\int d^4x \Big\{ \frac{\delta\overline{\Gamma}^{[1]}}{\delta K^a_\mu(x)} \frac{\delta\overline{\Gamma}^{[1]}}{\delta A_{a\mu}(x)} + \frac{\delta\overline{\Gamma}^{[1]}}{\delta L_a(x)} \frac{\delta\overline{\Gamma}^{[1]}}{\delta C_a(x)} \Big\} = 0\,. \tag{3.34}$$

$$\partial_\mu \frac{\delta\overline{\Gamma}^{[1]}}{\delta K^a_\mu(x)} - \frac{\delta\overline{\Gamma}^{[1]}}{\delta\overline{C}_a(x)} = 0\,. \tag{3.35}$$

We now know well how to prove the renormalisability of the theory by using the Slavnov–Taylor identities and the inductive method. Let us assume that up to n loop the theory has been proved to be renormalisable by introducing the counterterm

$$I^{[n]}_{count} = \sum_{l=1}^{n} \delta I^{[l]}_{count}$$

where $\delta I^{[l]}_{count}$ is the counterterm of order $\hbar^l$ and has the form of (3.34),(3.35). This also means that $\overline{\Gamma}^{[n]}$ determined by the action

$$I^{[n]}_{eff} = I^{[0]}_{eff} + I^{[n]}_{count}$$

satisfies the Slavnov–Taylor identities and is finite to order $\hbar^n$. We have to proved that by adding a counterterm of order $\hbar^{n+1}$ which also has the form of (3.34),(3.35), $\overline{\Gamma}^{[n+1]}$ determined by the action

$$I^{[n+1]}_{eff} = I^{[n]}_{eff} + \delta I^{[n+1]}_{count}$$

can be make satisfy the Slavnov–Taylor identities and finite to order $\hbar^{n+1}$.

Denote by $\overline{\Gamma}^{[n]}_k$ the part of order $\hbar^k$ in $\overline{\Gamma}^{[n]}$. For $k \le n$, $\overline{\Gamma}^{[n]}_k$ is equal to $\overline{\Gamma}^{[k]}_k$, because it can not contain the contribution of a counterterm of order $\hbar^{k+1}$ or higher. Thus on expanding $\overline{\Gamma}^{[n]}$ to order $\hbar^{n+1}$ one has

$$\overline{\Gamma}^{[n]} = \sum_{k=0}^{n} \overline{\Gamma}^{[k]}_k + \overline{\Gamma}^{[n]}_{n+1} + \cdots\,.$$

Using this and extracting the terms of order $\hbar^{(n+1)}$ in the Slavnov–Taylor identities of $\overline{\Gamma}^{[n]}$, we find

$$\overline{\Gamma}^{[0]}_0 * \overline{\Gamma}^{[n]}_{n+1} + \overline{\Gamma}^{[n]}_{n+1} * \overline{\Gamma}^{[0]}_0 = 0\,, \tag{3.36}$$

$$\partial_\mu \frac{\delta \overline{\Gamma}^{[n]}_{n+1}}{\delta K^a_\mu(x)} - \frac{\delta \overline{\Gamma}^{[n]}_{n+1}}{\delta \overline{C}_a(x)} = 0\,. \tag{3.37}$$

Let $\overline{\Gamma}^{[n]}_{n+1,\mathrm{div}}$ stand for the pole part of $\overline{\Gamma}^{[n]}_{n+1}$. By repeating the steps going from (3.21) to (3.23), one can arrive at

$$\begin{aligned}\overline{\Gamma}^{[n]}_{n+1,\mathrm{div}} &= \alpha_{n+1}\Big(g\frac{\partial \overline{\Gamma}^{[0]}_0}{\partial g}\Big) + \beta_{n+1}\int d^4x \Big\{ A_{a\nu}(x)\frac{\delta \overline{\Gamma}^{[0]}_0}{\delta A_{a\nu}(x)} + L_a(x)\frac{\delta \overline{\Gamma}^{[0]}_0}{\delta L_a(x)}\Big\} \\ &+\gamma_{n+1}\int d^4x \Big\{ C_a(x)\frac{\delta \overline{\Gamma}^{[0]}_0}{\delta C_a(x)} + \overline{C}_a(x)\frac{\delta \overline{\Gamma}^{[0]}_0}{\delta \overline{C}_a(x)} + K^a_\mu(x)\frac{\delta \overline{\Gamma}^{[0]}_0}{\delta K^a_\mu(x)}\Big\}\,. \end{aligned}\tag{3.38}$$

where α_{n+1}, β_{n+1} and γ_{n+1} are constants of order $(\hbar)^{n+1}$. Therefore, in order to cancel the $n+1$ loop divergence the counterterm of order $\hbar^{n+1}$ should be chosen as

$$\delta I^{[n+1]}_{count}[A, C, \overline{C}, K, L, g, M] = -\overline{\Gamma}^{[n]}_{n+1,\mathrm{div}}[A, C, \overline{C}, K, L, g, M]\,. \tag{3.39}$$

After adding this counterterm and the gauge fixing term as well as the mass term of the gauge feilds $I^{[n+1]}_{\mathrm{eff}}$, to order $\hbar^{n+1}$, can be expressed as

$$I^{[n+1]}_{\mathrm{eff}}[A, C, \overline{C}, K, L, g, M, \xi] = I^{[0]}_{\mathrm{eff}}[A^{[0]}, C^{[0]}, \overline{C}^{[0]}, K^{[0]}, L^{[0]}, g^{[0]}, M^{[0]}, \xi^{[0]}]\,, \tag{3.40}$$

where $A^{[0]}, C^{[0]}, \overline{C}^{[0]}, \cdots$, to order $\hbar^{n+1}$, stand for the bare quantities

$$A^{[0]}_{a\mu} = (Z^{[n+1]}_3)^{1/2} A_{a\mu}, \quad C^{[0]}_a = (\widetilde{Z}^{[n+1]}_3)^{1/2} C_a, \quad \overline{C}^{[0]}_a = (\widetilde{Z}^{[n+1]}_3)^{1/2}\overline{C}_a\,, \tag{3.41}$$

$$K^{[0]a}_\mu = (\widetilde{Z}^{[n+1]}_3)^{1/2} K^a_\mu, \quad L^{[0]}_a = (Z^{[n+1]}_3)^{1/2} L_a\,, \tag{3.42}$$

$$g^{[0]} = Z^{[n+1]}_g g, \quad M^{[0]} = (Z^{[n+1]}_3)^{-1/2} M, \quad \xi^{[0]} = Z^{[n+1]}_3 \xi\,. \tag{3.43}$$

with

$$\begin{aligned}(Z^{[n+1]}_3)^{1/2} &= (Z^{[n]}_3)^{1/2} - \beta_{n+1}\,,\\ (\widetilde{Z}^{[n+1]}_3)^{1/2} &= (\widetilde{Z}^{[n]}_3)^{1/2} - \gamma_{n+1}\,,\\ Z^{[n+1]}_g &= Z^{[n]}_g - \alpha_{n+1}\,.\end{aligned}$$

Therefore, the generating functiomal $\overline{\Gamma}^{[n+1]}$ for proper functions determined by the action $I^{[n+1]}_{eff}$ can be found from $\overline{\Gamma}^{[0]}$. Namely

$$\overline{\Gamma}^{[n+1]}[A, \overline{C}, C, K, L] = \overline{\Gamma}^{[0]}[A^{[0]}, \overline{C}^{[0]}, C^{[0]}, K^{[0]}, L^{[0]}]\,. \tag{3.44}$$

With this, one can verify that $\overline{\Gamma}^{[n+1]}$ satisfies (3.34),(3.35) and is finite to order $\hbar^{n+1}$. Since the theory can be renormalized to one loop the renormalisability has been proved by the inductive method.

IV. Renormalisability of the theory with variables λ_a

Similar to section 3, let $A_{a\mu}(x)$, $C_a(x)$ and $\overline{C}_a(x)$ stand for the renormalized feild founctions, g, M be renormalized parameters. The matter feilds ψ are also omitted. Now the effective Lagranian without counterterm is

$$\begin{aligned}\mathcal{L}^{[0]}_{\rm eff}(x) = & - \frac{1}{4}F_{a\mu\nu}(x)F^{\mu\nu}_a(x) + \frac{1}{2}M^2 A_{a\mu}(x)A^\mu_a(x) + \lambda_a(x)\partial^\nu A_{a\nu}(x) \\ & + \big(-\partial_\mu \overline{C}_a(x)\big)D^\mu_{ab}C_b(x) \\ & + K^a_\mu(x)\Delta A^\mu_a(x) + L_a(x)\Delta C_a(x)\,. \end{aligned} \tag{4.1}$$

The generating functional for Green functions is

$$\mathcal{Z}^{[0]}[J,j,\overline{\chi},\chi,K,L] = \frac{1}{N}\int \mathcal{D}[\mathcal{A},\overline{C},C,\lambda]\exp\Big\{{\rm i}\big(I^{[0]}_{\rm eff}+I_s\big)\Big\}\,, \tag{4.2}$$

where N is a constant to make $\mathcal{Z}^{[0]}[0,0,0,0,0,0]$ equal to 1, the source term I_s is given by

$$I_s = \int d^4x\big[J^\mu_a(x)A_{a\mu}(x) + j_a(x)\lambda_a(x) + \overline{\chi}_a(x)C_a(x) + \overline{C}_a(x)\chi_a(x)\big]\,,$$

Correspondingly, the generating functionals $\mathcal{W}^{[0]}$, $\Gamma^{[0]}$ for connected Green functions and regular vertex functions are

$$\mathcal{Z}^{[0]}[J,j,\overline{\chi},\chi,K,L] = \exp\Big\{{\rm i}\mathcal{W}^{[0]}[J,j,\overline{\chi},\chi,K,L]\Big\}\,, \tag{4.3}$$

$$\begin{aligned}\Gamma^{[0]}[\widetilde{A},\widetilde{\overline{C}},\widetilde{C},\widetilde{\lambda},K,L] = & \,\mathcal{W}^{[0]}[J,j,\overline{\chi},\chi,K,L] \\ & - \int d^4x\Big[J^\mu_a(x)\widetilde{A}_{a\mu}(x) + j_a(x)\widetilde{\lambda}_a(x) + \overline{\chi}_a(x)\widetilde{C}_a(x) + \widetilde{\overline{C}}_a(x)\chi_a(x)\Big]\,, \end{aligned} \tag{4.4}$$

where the classical fields are defined by

$$\widetilde{A}_{a\mu}(x) = \frac{\delta\mathcal{W}^{[0]}}{\delta J^\mu_a(x)}\,,\qquad \widetilde{\lambda}_a(x) = \frac{\delta\mathcal{W}^{[0]}}{\delta j_a(x)}\,,\qquad \widetilde{C}_a(x) = \frac{\delta\mathcal{W}^{[0]}}{\delta\overline{\chi}_a(x)}\,,\qquad \widetilde{\overline{C}}_a(x) = -\frac{\delta\mathcal{W}^{[0]}}{\delta\chi_a(x)}\,. \tag{4.5}$$

Therefore one has

$$J^\mu_a(x) = -\frac{\delta\Gamma^{[0]}}{\delta\widetilde{A}_{a\mu}(x)}\,,\qquad j_a(x) = -\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\lambda}_a(x)}\,,\qquad \overline{\chi}_a(x) = \frac{\delta\Gamma^{[0]}}{\delta\widetilde{C}_a(x)}\,,\qquad \chi_a(x) = -\frac{\delta\Gamma^{[0]}}{\delta\widetilde{\overline{C}}_a(x)}\,, \tag{4.6}$$

and

$$\frac{\delta \mathcal{W}^{[0]}}{\delta K^a_\mu(x)} = \frac{\delta \Gamma^{[0]}}{\delta K^a_\mu(x)}\,, \qquad \frac{\delta \mathcal{W}^{[0]}}{\delta L_a(x)} = \frac{\delta \Gamma^{[0]}}{\delta L_a(x)}\,, \tag{4.7}$$

In order to find the Slavnov–Taylor identity satisfied by the generating functional for the proper functions, we change the variables in the path integral of $\mathcal{Z}^{[0]}$ as follows

$$\begin{aligned}
&A^\mu_a(x) \to A^\mu_a(x) + \delta\zeta \Delta A^\mu_a(x)\,,\\
&C_a(x) \to C_a(x) + \delta\zeta \Delta C_a(x)\,,\\
&\overline{C}_a(x) \to \overline{C}_a(x) - \delta\zeta \lambda_a(x)\,,\\
&\lambda_a(x) \to \lambda_a(x)\,.
\end{aligned}$$

The volume element of the path integral does not change and the changes of the source term and the mass term of the gauge fields lead to

$$\begin{aligned}
\int d^4x \Big\{ &\frac{\delta \Gamma^{[0]}}{\delta K^a_\mu(x)} \frac{\delta \Gamma^{[0]}}{\delta \widetilde{A}^\mu_a(x)} + \frac{\delta \Gamma^{[0]}}{\delta L_a(x)} \frac{\delta \Gamma^{[0]}}{\delta \widetilde{C}_a(x)} \\
&-\widetilde{\lambda}_a(x) \frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} - M^2 \widetilde{A}_{a\mu}(x) \frac{\delta \Gamma^{[0]}}{\delta K^a_\mu(x)} \Big\} = 0\,,
\end{aligned} \tag{4.8}$$

Next, by using the invariance of the path integral of $\mathcal{Z}^{[0]}$ with respect to the translation of the integration variables $\overline{C}_a(x)$ and $\lambda_a(x)$, one can get a set of auxiliary identities

$$\partial_\mu \frac{\delta \Gamma^{[0]}}{\delta K^a_\mu(x)} - \frac{\delta \Gamma^{[0]}}{\delta \widetilde{\overline{C}}_a(x)} = 0\,, \tag{4.9}$$

$$\frac{\delta \Gamma^{[0]}}{\delta \widetilde{\lambda}_a(x)} - \partial^\mu \widetilde{A}_{a\mu}(x) = 0\,. \tag{4.10}$$

Let $\Gamma^{[0]}[A, \overline{C}, C, \lambda, K, L]$ be the functional that is obtained from $\Gamma^{[0]}[\widetilde{A}, \widetilde{\overline{C}}, \widetilde{C}, \widetilde{\lambda}, K, L]$ by replacing the classical field functions with the usual field functions. Defined $\overline{\Gamma}^{[0]}$ as

$$\overline{\Gamma}^{[0]} = \Gamma^{[0]} - \int d^4x \Big\{ \lambda_a(x) \partial^\mu A_{a\mu}(x) \Big\} - \int d^4x \Big\{ \frac{1}{2} M^2 A^\mu_a(x) A_{a\mu}(x) \Big\}\,, \tag{4.11}$$

Thus from (4.8)–(4.10) we have

$$\int d^4x \Big\{ \frac{\delta \overline{\Gamma}^{[0]}}{\delta K^a_\mu(x)} \frac{\delta \overline{\Gamma}^{[0]}}{\delta A^\mu_a(x)} + \frac{\delta \overline{\Gamma}^{[0]}}{\delta L_a(x)} \frac{\delta \overline{\Gamma}^{[0]}}{\delta C_a(x)} \Big\} = 0\,, \tag{4.12}$$

$$\partial_\mu \frac{\delta \overline{\Gamma}^{[0]}}{\delta K^a_\mu(x)} - \frac{\delta \overline{\Gamma}^{[0]}}{\delta \overline{C}_a(x)} = 0\,. \tag{4.13}$$

$$\frac{\delta \overline{\Gamma}^{[0]}}{\delta \lambda_a(x)} = 0\,. \tag{4.14}$$

align It is now obvious that the method used in last section can be followed to prove the renormalisability of the theory.

V. Concluding Remarks

We have expounded that the quantization under the Lorentz condition caused by the mass term of the gauge fields leads to a ghost action which is the same as that of the usual SU(n) theory in the Lorentz gauge. We have also shown that when the theory is generalized by regarding as new variables the Lagrange multipliers $\lambda_a(x)$ associated with the Lorentz condition, the BRST invariance is kept in the total effective action consisting of λ_a, ghost fields and the original fields. As a result of the mass term of the gauge fields, such a kind of BRST transformation is not nilpotent and can change the volume element of the path integral.

Furthermore, we have clarified that the mass term of the gauge fields cause no extra complexity to the Slavnov-Taylor identity of the generating functional for the regular vertex functions. In particular, the equations satisfied by the divergent part of this generating functional are independent of M. Consequently, we have been able to determine the general form of the counterterms order by order based on the renormalisability of the usual SU(n) gauge theory and prove that the mass term of the gauge fields is hurmless to the renormalisability of the theory. In this way we have also reveal that the renormalisability of the theory with the mass term of the gauge fields is ensured by that of the usual SU(n) theory.

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